Stolen Base Physics

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Few plays in baseball are as consistently close and exciting as the stolen base.[1] While there are several studies of sprinting,[2-4] the art of base stealing is much more nuanced. This article describes the motion of the base-stealing runner using a very basic kinematic model. The model will be compared to some data from a Major League game. The predictions of the model show consistency with the skills needed for effective base stealing.

The basic kinematic model

Let's just consider a steal of second base as opposed to third or home. The goal of the runner is to minimize the time required to get there. The basic kinematic model breaks the total distance between the bases ($D = 90.0$ ft) into four parts.

1. ***The lead off*** — the distance d_L away from first base where the steal attempt begins.
2. ***The jump*** — the distance d_j while accelerating at a constant rate of a_+ away from first base when starting from rest.
3. ***The run*** — the distance d_r while traveling at a constant top speed v.
4. ***The slide*** — the distance d_s while decelerating from top speed at a constant rate of a_- into second base at a velocity of v_f.

In summary, the model assumes the velocity-versus-time graph shown in Fig. 1, resulting in the distance-versus-time graph of Fig. 2. The total distance between the bases is just the sum of the four distances described above,

$$D = d_L + d_j + d_r + d_s. \tag{1}$$

We can use the kinematic equations to express the jump distance and the time for the jump in terms of the top speed v and the acceleration a_+,

$$d_j = \frac{v^2}{2a_+} \quad \text{and} \quad t_j = \frac{v}{a_+}. \tag{2}$$

Similarly, for the slide distance but including the final speed,

$$d_s = \frac{v^2 - v_f^2}{2a_-} \quad \text{and} \quad t_s = \frac{v - v_f}{a_-}. \tag{3}$$

The run distance needs to be expressed in terms of the total distance while the run time is just the distance over the velocity,

$$d_r = D - d_L - d_j - d_s \quad \text{and}$$
$$t_r = \frac{d_r}{v} = \frac{D - d_L - d_j - d_s}{v}. \tag{4}$$

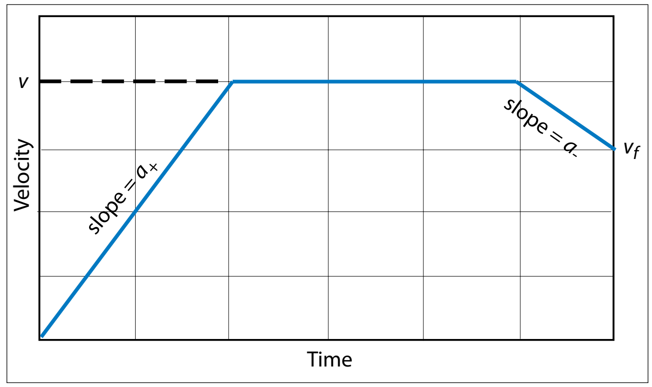

Fig. 1. The velocity-time graph for the kinematic model. It is the shape of the curve that describes the kinematic model, so units are omitted.

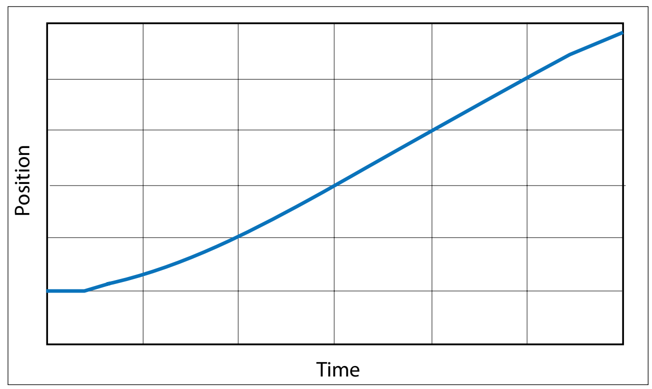

Fig. 2. The resulting position-time graph from the velocity-time graph of the kinematic model. Units are again omitted. Note the time axis is shifted from Fig. 1 due to the lead-off distance.

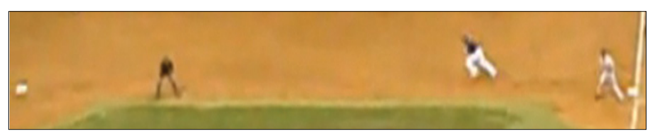

Fig. 3. A frame from the video of Crawford's stolen base.

The total time to steal second is

$$T = t_j + t_r + t_s = \frac{v}{a_+} + \frac{D - d_L - d_j - d_s}{v} + \frac{v - v_f}{a_-}. \tag{5}$$

Using the distance equations from (1), (2), and (3) plus a bit of algebra gives

$$T = \frac{v}{2a_+} + \frac{D - d_L}{v} + \frac{(v - v_f)^2}{2va_-}. \tag{6}$$

The total time depends upon five parameters and the distance between the bases:

a_+ = the acceleration away from first base,
a_- = the deceleration sliding into second base,
v = the top speed of the runner,
v_f = the speed of the runner upon reaching second base, and
d_L = the lead-off distance from first base.

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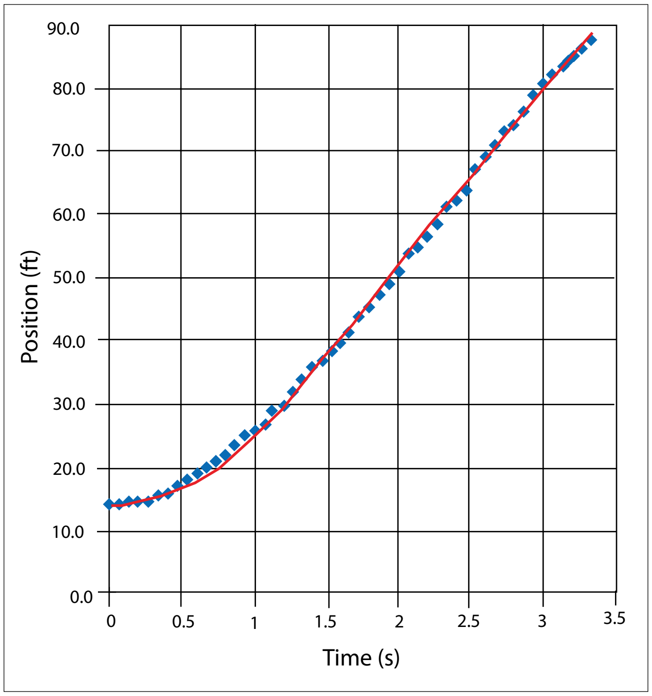

Fig. 4. The position vs time for the Crawford stolen base. The red line is a fit based on the kinematic model.

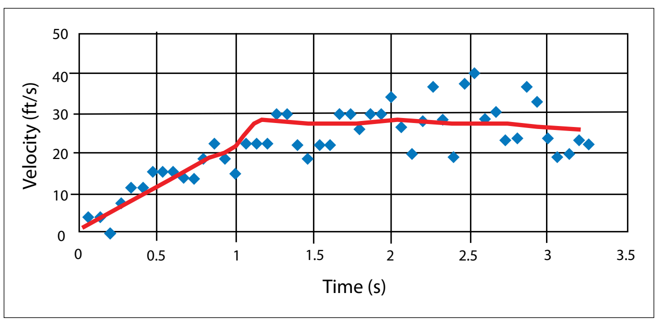

Fig. 5. The velocity vs time for the Crawford stolen base. The red line is a fit based on the kinematic model.

Equation (6) is anything but transparent. However, we'll address the importance of each parameter in a bit after we get some typical data from an actual Major League stolen base.

Some actual data

Getting position-versus-time data for a stolen base should be relatively easy. Just find some video, use the 90 feet between the bases as a scale, measure the position of the runner frame by frame, and you're done. Not so fast! If you watch a game on TV you will rarely find a shot that watches the runner from beginning to end. If you do, it is very likely that the camera either moves or zooms during the play. This destroys the scaling. So, professional video of a game is rarely helpful.

This is where YouTube, a fan in the cheap seats, and baseball history all come together for the benefit of physics. The record for stolen bases in a single Major League game is six. On May 3, 2009, Carl Crawford of the Tampa Bay Rays had already stolen five when he reached first in the eighth inning. A fan in the top deck high above the third-base line expectantly turned on a video camera hoping to capture the historic moment when the record would be tied. Crawford indeed stole second.[5] Our unnamed but nonetheless beloved fan posted the video on YouTube.[6] Figure 3 is an image from that video.

Figure 4 is a plot of position versus time for the Crawford stolen base data. The blue diamonds are the data points. The red line has the same shape as Fig. 2 because it is a least-squares fit to this data using the kinematic model. The values of the parameters for this fit are:

$a_+ = 21.1$ ft/s^2
$a_- = 3.75$ ft/s^2
$v = 28.1$ ft/s
$v_f = 25.5$ ft/s
$d_L = 13.8$ ft.

You might notice that between 0.25 s and 1.25 s, the data points are consistently above the best fit. This is due to modeling the acceleration as constant when careful analysis of sprinters shows that the acceleration drops slowly with time.[2-4]

Figure 5 is the velocity versus time. The velocity values fluctuate due to the propagation of uncertainties from the position values. Therefore, little can be said about the validity of the kinematic model from the velocity other than the red line, which has the same shape as Fig. 1, is not inconsistent with the data.

Lessons from the kinematic model

The kinematic model should have some insight into the best way to go about stealing bases. Since Eq. (6) is so intractable, one way to proceed is to find the sensitivity to each of the five parameters by differentiating the total time of Eq. (6) with respect to each one,

$$\frac{\partial T}{\partial a_+} = -\frac{v}{2a_+^2} \Rightarrow \Delta T = -\frac{v}{2a_+}\frac{\Delta a_+}{a_+} \tag{7}$$

$$\frac{\partial T}{\partial a_-} = -\frac{\left(v - v_\mathrm{f}\right)^2}{2va_-^2} \Rightarrow \Delta T = -\frac{\left(v - v_\mathrm{f}\right)^2}{2va_-}\frac{\Delta a_-}{a_-}\ , \tag{8}$$

$$\frac{\partial T}{\partial v} = \frac{1}{v}\left(\frac{v}{2a_+} - \frac{D - d_\mathrm{L}}{v} + \frac{v^2 - v_\mathrm{f}^2}{2a_- v}\right) \Rightarrow$$
$$\Delta T = \left(\frac{v}{2a_+} - \frac{D - d_\mathrm{L}}{v} + \frac{v^2 - v_\mathrm{f}^2}{2a_- v}\right)\frac{\Delta v}{v}, \tag{9}$$

$$\frac{\partial T}{\partial v_\mathrm{f}} = -\frac{\left(v - v_\mathrm{f}\right)}{va_-} = -\frac{1}{v_\mathrm{f}}\frac{v_\mathrm{f}}{va_-}\left(v - v_\mathrm{f}\right) \Rightarrow$$
$$\Delta T = -\frac{v_\mathrm{f}}{va_-}\left(v - v_\mathrm{f}\right)\frac{\Delta v_\mathrm{f}}{v_\mathrm{f}}, \tag{10}$$

$$\text{and} \quad \frac{\partial T}{\partial d_{\mathrm{L}}} = -\frac{1}{v} \Rightarrow \Delta T = -\frac{d_{\mathrm{L}}}{v}\frac{\Delta d_{\mathrm{L}}}{d_{\mathrm{L}}}. \tag{11}$$

These equations can be made a bit more helpful by plugging in the fitted values of the parameters to get a sense of the size of each contribution. The results are

$$\Delta T = -(0.67\ \mathrm{s})\frac{\Delta a_+}{a_+}, \tag{12}$$

$$\Delta T = -(0.032\ \mathrm{s})\frac{\Delta a_-}{a_-}, \tag{13}$$

$$\Delta T = -(1.4\ \mathrm{s})\frac{\Delta v}{v}, \tag{14}$$

$$\Delta T = -(0.63)\frac{\Delta v_{\mathrm{f}}}{v_{\mathrm{f}}}, \tag{15}$$

$$\text{and } \Delta T = -(0.49\ \mathrm{s})\frac{\Delta d_{\mathrm{L}}}{d_{\mathrm{L}}}. \tag{16}$$

Fractional changes in top speed shorten the time the most, followed by the acceleration during the jump, the final speed at second base, the size of the lead off, and finally the deceleration rate near second base, which is essentially insignificant.

I believe most baseball people would agree that the two biggest keys to success are the top speed of the runner and the acceleration away from first base. This explains why only the fleetest of foot even attempt to steal a base.

Some would disagree with the conclusion that the speed of arrival at second base is more important than the distance of the lead off. Yet, the author has noticed that nowadays many base stealers try to over-slide the base intentionally. They use the base itself to stop by grabbing it with a hand or a foot on the way, thus maintaining the highest possible speed when initially touching second base. So, perhaps this result is consistent with the behavior of professional ballplayers.

The speed upon reaching second base is only slightly more important than the lead-off distance. Announcers often focus heavily on the size of the lead, giving the impression that it is of central importance. According to this model, a big lead off might indicate that a steal will be attempted, but it is not the most important factor in determining success.

References

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